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Crosscap states and Boundary states in $D = 4, N = 1$,type-IIB Orientifold Theories

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Abstract

We construct boundary state and crosscap state in $D = 4, N = 1$ type-IIB Z_N orientifold and investigate properties of amplitude. We find that the boundary state of a cylinder is different from the boundary state of a Möbius strip. Using these states, we find that amplitudes do not factorize in $Z_N(N =\text{even})$ orientifold. Tadpole divergence remain in Z_4, Z_8, Z_8' and Z_{12}' model due to volume dependence of boundary and crosscap state. On the other hand the amplitude of Z_3 and Z_7 orientifolds factorize so that we obtain the gauge groups of the model by employing the massless tadpole cancellation condition.

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1 Introduction

The boundary state formalism was introduced to interprete open string amplitudes in terms of closed string states. This could be valuable in understanding the relationship between closed and open strings which is one of the central problems in uncovering the underlying symmetry of string theory. It serves very useful in analyzing the spectrum of models that do not have an obvious geometrical interpretation such as orbifold with discrete torsion [2]. Recently, boundary state formalism in conformal field theory is explored [3][4][5]. The boundary state formalism is a powerful framework for studying D-branes [6][7], and useful for computing D-brane tensions and cylinder amplitudes as well as in looking for the gravity counterparts of D-branes. The structure of $D = 4, N = 1$, type-IIB orientifold is explored, and conditions for tadpole cancellation in type-IIB Z_N orientifold have been presented [8]. However, in spite

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of importance to understand the relationship between closed and open string, the relation is not investigated in $D=4, N=1$, type-IIB orientifold.

In this paper, we derive crosscap states and boundary states for D-branes at a fixed point in $D=4, N=1$, type-IIB Z_N orientifold theory. We find that a boundary state in Möbius strip is different from that in cylinder in general. And we investigate the structure of factorization in Z_N orbifolds. Even though amplitude factorizes in $D=10$ models in general, amplitude does not factorize in general in $D=4, Z_N(N=$even$)$ models. In section 3 we consider momentum and winding modes of boundary states and crosscap states. The tadpole divergence remains in Z_4, Z_8, Z_8' and Z_{12}' model. Using tadpole cancellation condition, we derive gauge groups in Z_3, Z_7 models.

2 Construction of crosscap and boundary states

At first, we construct a crosscap state $|C\rangle$ in $D=4, N=1$, type-IIB Z_N orientifold model. We summarize in Table 1 the Z_N action that leads to $N=1$ supersymmetry .

The mode expansion of a closed string state reads

$$X^\mu(\sigma_1,\sigma_2) = x^\mu + l^2 p^\mu \sigma_1 + \frac{i}{2} l \sum_{n\neq 0} (\frac{1}{n}\alpha_n^\mu e^{-2i\pi n(\sigma_2-\sigma_1)} + \frac{1}{n}\tilde{\alpha}_n^\mu e^{-2i\pi n(\sigma_2+\sigma_1)}). \tag{2.1}$$

We introduce the complex coordinate,

$$Y^i = X^{2i+2} + iX^{2i+3}, \quad \bar{Y}^i = X^{2i+2} - iX^{2i+3}, \quad (i=1,2,3). \tag{2.2}$$

The Z_N orientifold model has k-twisted sectors of the closed string. The boundary condition for the Klein bottle is

$$Y^i(\sigma_1+1,\sigma_2) = e^{2i\pi k v_i} Y^i(\sigma_1,\sigma_2) \qquad\qquad (i=1,2,3), \tag{2.3}$$

with a fundamental domain $0 \le \sigma_1 < 1$ and $0 \le \sigma_2 < 1$ in Figure 1. By using another fundamental domain $0 \le \sigma_1 < \frac{1}{2}$ and $0 \le \sigma_2 < 2$, the boundary condition for the Klein bottle is

$$Y^i(0,\sigma_2+2) = e^{4i\pi k v_i} Y^i(0,\sigma_2), \qquad Y^i(\frac{1}{2},\sigma_2+2) = Y^i(\frac{1}{2},\sigma_2). \tag{2.4}$$

It means Y^i at $\sigma_1 = 0$ has $2k$-twisted sectors, while Y^i at $\sigma_1 = \frac{1}{2}$ is not twisted.

The state produced from the vacuum by crosscap is determined up to normalization by crosscap conditions on the fields. Here we define $\tau = 2\sigma_1, \sigma = \frac{1}{2}\sigma_2$. And we denote Y^i at $\sigma_1 = 0$, $Y_0^i(\sigma)$, and at $\sigma_1 = \frac{1}{2}$, $Y^i_{\frac{1}{2}}(\sigma)$. $Y^i_{0,\frac{1}{2}}(\sigma)$ has $2m$-twisted sectors. Then

$$Y_0^i(\sigma+1) = e^{4i\pi(kv_i+mv_i)} Y_0^i(\sigma), \qquad Y^i_{\frac{1}{2}}(\sigma+1) = e^{4i\pi m v_i} Y^i_{\frac{1}{2}}(\sigma). \tag{2.5}$$

We denote $2k$-twisted, $2m$-twisted crosscap state by $|c,2kv_i,2mv_i\rangle^i$, where v_i is a component of the twist vector (v_1,v_2,v_3) in Table 1. The crosscap state conditions read

$$\begin{aligned}
\Big(Y_0^i(\sigma+\frac{1}{2})-e^{2i\pi(kv_i+mv_i)}Y_0^i(\sigma)\Big)|c,2kv_i,2mv_i\rangle^i &= 0,\\
\Big(\partial_\tau Y_0^i(\sigma+\frac{1}{2})+e^{2i\pi(kv_i+mv_i)}\partial_\tau Y_0^i(\sigma)\Big)|c,2kv_i,2mv_i\rangle^i &= 0,\\
\Big(Y_{\frac{1}{2}}^i(\sigma+\frac{1}{2})-e^{2i\pi mv_i}Y_{\frac{1}{2}}^i(\sigma)\Big)|c,0,2mv_i\rangle^i &= 0,\\
\Big(\partial_\tau Y_{\frac{1}{2}}^i(\sigma+\frac{1}{2})+e^{2i\pi mv_i}\partial_\tau Y_{\frac{1}{2}}^i(\sigma)\Big)|c,0,2mv_i\rangle^i &= 0.
\end{aligned} \tag{2.6}$$

By solving the conditions(2.6), the crosscap states are

$$\begin{aligned}
|c,2kv_i,2mv_i\rangle^i &= \prod_{n=1}\exp[\frac{-e^{-i\pi(n-1)}\bar{\beta}^i_{-n+1-2(k+m)v_i}\tilde{\beta}^i_{-n+1-2(k+m)v_i}}{n-1+2(k+m)v_i}]\\
&\quad \exp[\frac{-e^{i\pi n}\beta^i_{-n+2(k+m)v_i}\tilde{\bar{\beta}}^i_{-n+2(k+m)v_i}}{n-2(k+m)v_i}]|0\rangle,
\end{aligned} \tag{2.7}$$

where $|0\rangle$ is the Fock vacuum and β^i_n are oscillator modes of Y^i defined by

$$\begin{aligned}
\beta^i_n = &\ \alpha_n^{2i+2}+i\alpha_n^{2i+3}, \quad \bar{\beta}^i_n = \alpha_n^{2i+2}-i\alpha_n^{2i+3},\\
\tilde{\beta}^i_n = &\ \tilde{\alpha}_n^{2i+2}+i\tilde{\alpha}_n^{2i+3}, \quad \tilde{\bar{\beta}}^i_n = \tilde{\alpha}_n^{2i+2}-i\tilde{\alpha}_n^{2i+3} \quad (i=1,2,3).
\end{aligned} \tag{2.8}$$

Next we consider fermionic parts. Fermionic parts of the closed string are

$$\psi^\mu(\sigma_1,\sigma_2)=\sqrt{2\pi}\sum_r\psi_r^\mu e^{-2i\pi r(\sigma_2-\sigma_1)}, \quad \tilde{\psi}^\mu(\sigma_1,\sigma_2)=\sqrt{2\pi}\sum_r\tilde{\psi}_r^\mu e^{-2i\pi r(\sigma_2-\sigma_1)}. \tag{2.9}$$

Similar conditions as the bosonic case lead to the crosscap states,

$$\begin{aligned}
|c,2kv_i,2mv_i\rangle^i &= \prod_{n=1}\exp[i\eta e^{-i\pi n}\bar{\lambda}^i_{-n+1-2(k+m)v_i}\tilde{\lambda}^i_{-n+1-2(k+m)v_i}]\\
&\quad \exp[-i\eta e^{-i\pi n}\lambda^i_{-n+2(k+m)v_i}\tilde{\bar{\lambda}}^i_{-n+2(k+m)v_i}]|0,\eta\rangle.
\end{aligned} \tag{2.10}$$

where $|0,\eta\rangle$ is a usual Ramond-Ramond vacuum $\eta=\pm1$ and

$$\begin{aligned}
\lambda^i_r = &\ \psi_r^{2i+2}+i\psi_r^{2i+3}, \qquad \bar{\lambda}^i_r = \psi_r^{2i+2}-i\psi_r^{2i+3},\\
\tilde{\lambda}^i_r = &\ \tilde{\psi}_r^{2i+2}+i\tilde{\psi}_r^{2i+3}, \qquad \tilde{\bar{\lambda}}^i = \tilde{\psi}_r^{2i+2}-i\tilde{\psi}_r^{2i+3}.
\end{aligned} \tag{2.11}$$

By combining the bosonic and fermionic contributions, the crosscap state are defined by

$$\begin{aligned}
|C,2k\rangle &= \sum_{m=0}^{N-1}N_{c,2k,2m}|c,2k,2m\rangle,\\
|c,2k,2m\rangle &= |c,2kv_3,2mv_3\rangle^3|c,2kv_2,2mv_2\rangle^2|c,2kv_1,2mv_1\rangle^1|c,0,0\rangle^0,
\end{aligned} \tag{2.12}$$

where $|c,0,0\rangle^0$ denotes the crosscap state for the uncompactified (X^2,X^3) plane. The coefficients $N_{c,2k,2m}$ will be determined by tadpole cancellation conditions. We denote especially $k=0$ crosscap state by $|C\rangle$.

Next we consider a boundary state $|B\rangle$. Boundary state of cylinder has already constructed in ref.[3]. The results are

$$\begin{aligned}
|b,0,mv_i\rangle^i_{DD} &= \prod_{n=1}\exp[\frac{\bar{\beta}^i_{-(n-1)-mv_i}\tilde{\bar{\beta}}^i_{-(n-1)-mv_i}}{n-1+mv_i}]\exp[\frac{\beta^i_{-n+mv_i}\tilde{\tilde{\beta}}^i_{-n+mv_i}}{n-mv_i}] \\
&\prod_{n=1}\exp[i\eta\bar{\lambda}^i_{-n+1-mv_i}\tilde{\bar{\lambda}}^i_{-n+1-mv_i}]\exp[i\eta\lambda^i_{-n+mv_i}\tilde{\tilde{\lambda}}^i_{-n+mv_i}]|0,\eta\rangle, \\
|b,0,mv_i\rangle^i_{NN} &= \prod_{n=1}\exp[\frac{-\bar{\beta}^i_{-(n-1)-mv_i}\tilde{\bar{\beta}}^i_{-(n-1)-mv_i}}{n-1+mv_i}]\exp[\frac{-\beta^i_{-n+mv_i}\tilde{\tilde{\beta}}^i_{-n+mv_i}}{n-mv_i}] \\
&\prod_{n=1}\exp[-i\eta\bar{\lambda}^i_{-n+1-mv_i}\tilde{\bar{\lambda}}^i_{-n+1-mv_i}]\exp[-i\eta\lambda^i_{-n+mv_i}\tilde{\tilde{\lambda}}^i_{-n+mv_i}]|0,\eta\rangle.
\end{aligned} \tag{2.13}$$

Here subscripts DD and NN mean Dirichlet and Neumann boundary conditions in X^{2i+2}, X^{2i+3} $(i=1,2,3)$ directions, respectively. D9-branes has Neumann boundary conditions in $X^\mu(\mu=1,\dots 9)$ directions. D5-branes has Neumann boundary conditions in $X^\mu(\mu=1,2,3,8,9)$ directions and Dirichlet boundary conditions in $X^\mu(\mu=4,5,6,7)$ directions. Hence the boundary states $|B^c\rangle$ for cylinder on D5/D9-branes that include the Chan-Paton factors are defined as

$$\begin{aligned}
|B^c\rangle_p &= \sum_{m=0}^{N-1} N^c_{b,0,m,p}|b,0,m\rangle_p(Tr\gamma_{m,p}) \qquad (p=5,9), \\
|b,0,m\rangle_5 &= |b,0,mv_3\rangle^3_{NN}|b,0,mv_2\rangle^2_{DD}|b,0,mv_1\rangle^1_{DD}|b,0,0\rangle^0_{NN}, \\
|b,0,m\rangle_9 &= |b,0,mv_3\rangle^3_{NN}|b,0,mv_2\rangle^2_{NN}|b,0,mv_1\rangle^1_{NN}|b,0,0\rangle^0_{NN}.
\end{aligned} \tag{2.14}$$

Coefficients $N^c_{b,0,m,p}$ will be determined by tadpole cancellation condition.

Next we consider Möbius strip. We have Möbius strip boundary conditions,

$$\begin{aligned}
&Y^i(1,\sigma_2)=Y^i(0,\sigma_2+1), && \partial_1 Y^i(0,\sigma_2)=0 \\
&Y^i(\frac{1}{2},\sigma_2)=Y^i(\frac{1}{2},\sigma_2+1), && \partial_1 Y^i(\frac{1}{2},\sigma_2)=-\partial_1 Y^i(\frac{1}{2},\sigma_2+1)
\end{aligned} \tag{2.15}$$

Similar consideration like Klein bottle leads to the conclusion that the boundary $Y^i_0(\sigma)$ and the crosscap $Y^i_{\frac{1}{2}}(\sigma)$ do not possess k-twisted sectors but have $2m$-twisted sectors:

$$Y^i_0(\sigma+1)=e^{4i\pi mv_i}Y^i_0(\sigma), \quad Y^i_{\frac{1}{2}}(\sigma+1)=e^{4i\pi mv_i}Y^i_{\frac{1}{2}}(\sigma). \tag{2.16}$$

We denote $2m$-twisted crosscap state by $|c,0,2mv_i\rangle^i$, and $2m$-twisted boundary state by $|b,0,2mv_i\rangle^i$. The crosscap states $|c,0,2mv_i\rangle^i$ are same as those for the Klein bottle. But boundary state $|b,0,2mv_i\rangle^i$ which is $2m$-twist is different from boundary state in cylinder $|b,0,mv_i\rangle^i$ which is m-twist. We determine boundary states $|B^M\rangle$ for Möbius strip that include the Chan-Paton factors.

$$\begin{aligned} |B^M\rangle_p &= \sum_{m=0}^{N-1} N^M_{b,0,2m,p}|b,0,2m\rangle_p(Tr\gamma_{2m,p}), \qquad (p=5,9), \\ |b,0,2m\rangle_5 &= |b,0,2mv_3\rangle^3_{NN}|b,0,2mv_2\rangle^2_{DD}|b,0,2mv_1\rangle^1_{DD}|b,0,0\rangle^0_{NN}, \\ |b,0,2m\rangle_9 &= |b,0,2mv_3\rangle^3_{NN}|b,0,2mv_2\rangle^2_{NN}|b,0,2mv_1\rangle^1_{NN}|b,0,0\rangle^0_{NN}. \end{aligned} \tag{2.17}$$

By using crosscap state (2.12), boundary states (2.14) and (2.17) and Cardy's condition [5][6], the amplitudes for Klein bottle, cylinder and Möbius strip are summarized as

$$\begin{aligned} \mathcal{K} &= \sum_I \int dl(\langle C_I|e^{-lH}|C_I\rangle + \langle C_I,N|e^{-lH}|C_I\rangle) = \frac{V_4}{N}\sum_{m=0}^{N-1}\int_0^\infty \frac{dt}{16t^3}\frac{\tilde\vartheta[^0_{\frac12}](2t)}{\tilde\eta^3_{(2t)}}(Z_K+Z_{KT}), \\ \mathcal{C}_{pq} &= \sum_I \int dl_p\langle B^c_I|e^{-lH}|B^c_I\rangle_q = \frac{V_4}{N}\sum_{m=0}^{N-1}\int_0^\infty \frac{dt}{4t^3}\frac{\vartheta[^0_{\frac12}](t)}{\eta^3_{(t)}}Z_{pq}, \\ \mathcal{M}_p &= \sum_I \int dl_p\langle B^M_I|e^{-lH}|C_I\rangle = \frac{V_4}{N}\sum_{m=0}^{N-1}\int_0^\infty \frac{dt}{32t^3}\frac{\tilde\vartheta[^0_{\frac12}](2t)\tilde\vartheta[^{\frac12}_0](2t)}{\tilde\eta^3_{(2t)}\tilde\vartheta[^0_0](2t)}Z_p. \end{aligned} \tag{2.18}$$

An index I stands for the fixed point. Explicit form of $Z_K, Z_{KT} Z_{pq}$ and Z_p are

$$\begin{aligned} Z_K &= \prod_{i=1}^3 \frac{-2\sin 2\pi m v_i \tilde\vartheta[^0_{2mv_i+\frac12}](2t)}{\tilde\vartheta[^{\frac12}_{2mv_i+\frac12}](2t)} N^2_{c,0,2m}, \\ Z_{KT} &= \prod_{i=1}^2 \frac{\tilde\vartheta[^{\frac12}_{2mv_i+\frac12}](2t)}{\tilde\vartheta[^0_{2mv_i+\frac12}](2t)} \frac{-2\sin 2\pi m v_3 \tilde\vartheta[^0_{2mv_3+\frac12}](2t)}{\tilde\vartheta[^{\frac12}_{2mv_3+\frac12}](2t)} N_{c,N,2m}N_{c,0,2m}, \\ Z_{55} &= \prod_{i=1}^3 \frac{-2\sin \pi m v_i \vartheta[^0_{mv_i+\frac12}](t)}{\vartheta[^{\frac12}_{mv_i+\frac12}](t)} \sum_I (Tr\gamma_{m,5,I})^2(N^c_{b,0,m,5})^2, \\ Z_{99} &= \prod_{i=1}^3 \frac{-2\sin \pi m v_i \vartheta[^0_{mv_i+\frac12}](t)}{\vartheta[^{\frac12}_{mv_i+\frac12}](t)} (Tr\gamma_{m,9})^2(N^c_{b,0,m,9})^2, \\ Z_{59} &= \frac{-2\sin \pi m v_3 \vartheta[^0_{mv_3+\frac12}](t)}{\vartheta[^{\frac12}_{mv_3+\frac12}](t)} \prod_{i=1,2} \frac{\vartheta[^{\frac12}_{mv_i+\frac12}](t)}{\vartheta[^0_{mv_i+\frac12}](t)} (Tr\gamma_{m,9})\sum_I (Tr\gamma_{m,5,I}) N^c_{b,0,m,9}N^c_{b,0,m,5}, \\ Z_9 &= \prod_{i=1}^3 \frac{-2\sin \pi m v_i \tilde\vartheta[^0_{mv_i+\frac12}](2t)\tilde\vartheta[^{\frac12}_{mv_i}](2t)}{\tilde\vartheta[^0_{mv_i}](2t)\tilde\vartheta[^{\frac12}_{mv_i+\frac12}](2t)} (Tr\gamma_{2m,9})N^M_{b,0,2m,9}N_{c,0,2m}, \\ Z_5 &= \frac{-2\sin \pi m v_3 \tilde\vartheta[^0_{mv_3+\frac12}](2t)\tilde\vartheta[^{\frac12}_{mv_3}](2t)}{\tilde\vartheta[^0_{mv_3}](2t)\tilde\vartheta[^{\frac12}_{mv_3+\frac12}](2t)} \end{aligned}$$

$$\prod_{i=1,2} \frac{2\cos \pi m v_i \tilde{\vartheta}[^{\frac{1}{2}}_{m v_i+\frac{1}{2}}](2t) \tilde{\vartheta}[^{0}_{m v_i}](2t)}{\tilde{\vartheta}[^{\frac{1}{2}}_{m v_i}](2t) \tilde{\vartheta}[^{0}_{m v_i+\frac{1}{2}}](2t)} \sum_I (Tr\gamma_{2m,5,I}) N^M_{b,0,2m,5} N_{c,0,2m}. \tag{2.19}$$

Note that Z_{KT} does vanish for Z_N(odd N). Comparing (2.19) with the amplitudes of the one-loop vacuum diagram [1], we obtain the following relations,:

$$\begin{aligned} (N^c_{b,0,m,5})^2 = \quad & (N^c_{b,0,m,9})^2 & = N^c_{b,0,m,5} N^c_{b,0,m,9} = \frac{1}{32\pi^4}, \\ N^2_{c,0,2m} = \quad & N_{c,0,2m} N_{c,N,2m} & = \frac{1}{2\pi^4}, \\ N_{c,0,2m} N^M_{b,0,2m,9} = \quad & N_{c,0,2m} N^M_{b,0,2m,5} & = \frac{-1}{4\pi^4}. \end{aligned} \tag{2.20}$$

As we have discussed, boundary states in Möbius strip and cylinder are different; $|B^c\rangle_p$ is the sum of m-twisted states but $|B^M\rangle_p$ is the sum of $2m$-twisted states. The amplitude for the Möbius strip is

$$\langle B^M | e^{-lH} | C \rangle = \sum_{m=0}^{N-1} N^M_{b,0,2m} N_{c,0,2m} \langle b, 0, 2m | e^{-lH} | c, 0, 2m \rangle (Tr\gamma_{2m}). \tag{2.21}$$

When we require a factorization of the total amplitude $(\langle B^c| + \langle C|) e^{-lH} (|B^c\rangle + |C\rangle)$, the Möbius strip amplitude is expressed using $|B^c\rangle$ as

$$\langle B^c | e^{-lH} | C \rangle + \langle C | e^{-lH} | B^c \rangle = 2 \sum_{m=0}^{N-1} N^c_{b,0,2m} N_{c,0,2m} \langle b, 0, 2m | e^{-lH} | c, 0, 2m \rangle (Tr\gamma_{2m}). \tag{2.22}$$

Therefore equality of (2.21) and (2.22) requires $N^M_{b,0,2m} = 2N^c_{b,0,2m}$. It leads

$$\begin{aligned} & N_{c,0,m} = N_{c,N,m} = -2N^M_{b,0,m,9} = -2N^M_{b,0,m,5} \\ & = -4N^c_{b,0,m,5} = -4N^c_{b,0,m,9} = \pm \frac{1}{\sqrt{2}\pi^2} \qquad (m = \text{even}), \end{aligned} \tag{2.23}$$

$$N^c_{b,0,m,5} = N^c_{b,0,m,9} = \pm \frac{1}{4\sqrt{2}\pi^2} \qquad (m = \text{odd}). \tag{2.24}$$

In the following, we use notation $|B\rangle$ for boundary states in Möbius strip and cylinder taking (2.21)(2.22)(2) into consideration.

Next let us examine the factorization of amplitude. It is generally believed that amplitude can be factorized. However it is not obvious in $N = 1, D = 4$ type-IIB Z_N orientifold. In the case of Z_3, Z_7 model

$$({}_9\langle B| + \langle C|) e^{-lH} (|B\rangle_9 + |C\rangle) = {}_9\langle B| e^{-lH} |B\rangle_9 + \langle C| e^{-lH} |C\rangle + 2\,{}_9\langle B| e^{-lH} |C\rangle$$

$$\mathcal{K} \sim \langle C| e^{-lH} |C\rangle, \quad \mathcal{M}_9 \sim 2\,{}_9\langle B| e^{-lH} |C\rangle, \quad \mathcal{C}_{99} \sim {}_9\langle B| e^{-lH} |B\rangle_9 \tag{2.25}$$

Hence amplitude does factorize. However in case of Z_6, the would-be factorized amplitude

$$({}_5\langle B| + {}_9\langle B| + \langle C|) e^{-lH} (|B\rangle_5 + |B\rangle_9 + |C\rangle) \tag{2.26}$$

contains the amplitudes $\mathcal{C}_{pp}$ and $\mathcal{M}_p$ but not all of the Klein bottle amplitude $\mathcal{K}$. Instead, if we include $\langle C, N|$ into the would-be factorized amplitude,

$$({}_5\langle B| +{}_9 \langle B| + \langle C| + \langle C, N|)e^{-lH}(|B\rangle_5 + |B\rangle_9 + |C\rangle + |C, N\rangle), \tag{2.27}$$

then three new amplitudes

$$\langle C, N|e^{-lH}|C, N\rangle, \quad 2_9\langle B|e^{-lH}|C, N\rangle, \quad 2_5\langle B|e^{-lH}|C, N\rangle. \tag{2.28}$$

do not possess geometric interpretation. This is a general situation in $Z_N(N=\text{even})$ model. Thus we conclude that the amplitude does not factorize in $Z_N(N = \text{even})$ model.

3 Volume dependence of boundary and crosscap states

We consider zero modes of boundary and crosscap states. Boundary state $|b, 0, mv_i\rangle^i$ is closed string so that it has momentum and winding in compactified space. The m-twisted boundary state $|b, 0, mv_i\rangle^i (mv_i \neq 0 \text{ mod } N)$ sits at fixed point so that it does not have momentum and winding. Hence boundary state which have momentum or winding is only $|b, 0, 0(\text{mod}N)\rangle^i$. When a cylinder has Dirichlet boundary condition in X^8 and X^9 directions, boundary closed string state can move to these directions and have momentum. When a cylinder has Neumann condition in X^8 and X^9 directions, open string can move to these directions and make loop in compactified direction. Hence boundary state have winding to these directions. Therefore $|b, 0, 0\rangle^i_{DD}$ has momentum and $|b, 0, 0\rangle^i_{NN}$ has winding. The crosscap state $|c, 2kv_i, 2mv_i\rangle^i$ can also have momentum and winding in compactified space. By the same argument, the crosscap state $|c, 2kv_i, 2mv_i\rangle^i$ has momentum or winding only if $2kv_i \equiv 0 \pmod N$ and $2mv_i \equiv 0 \pmod N$. To determine momentum and winding of $|c, 0, 0\rangle^i$, we consider Möbius strip amplitude ${}_{NN}{}^i\langle b, 0, 0|e^{-lH}|c, 0, 0(\text{mod}2N)\rangle^i$. Because ${}_{NN}{}^i\langle b, 0, 0|$ has winding, consistency requires that $|c, 0, 0(\text{mod}2N)\rangle^i$ has winding. In the same way $|c, 0, N\rangle^i$ has momentum because $|b, 0, N\rangle^i_{DD}$ has momentum. Properties of the zero modes for $|c, N(\text{mod}2N), 0(\text{mod}2N)\rangle^i$ and $|c, N(\text{mod}2N), N(\text{mod}2N)\rangle^i$ can not be obtained because ${}^i\langle c, N, 0|e^{-lH}|c, 0, 0\rangle^i$ and ${}^i\langle c, N, N|e^{-lH}|c, 0, N\rangle^i$ vanish.

Next we consider the volume dependence produced by momentum and winding. Hamiltonian which represents momentum and winding in compactified six dimensions is

$$H_2 = \pi(\frac{P}{2} - L)^2, \qquad \tilde{H}_2 = \pi(\frac{P}{2} + L)^2. \tag{3.1}$$

We denote boundary states which has momentum and winding as

$$\begin{aligned} |b, 0, 0\rangle^i &= |b, 0, 0; n, m\rangle^i = e^{i(\frac{n\cdot x}{R_i} + m\cdot xR_i)}|n = 0, m = 0\rangle^i, \\ |c, 0, 0\rangle^i &= |c, 0, 0; n, m\rangle^i = e^{i(\frac{n\cdot x}{2R_i} + \frac{m\cdot xR_i}{2})}|n = 0, m = 0\rangle^i, \end{aligned} \tag{3.2}$$

where R_i is a common radius of the i th compactified space. From these formula, we obtain the volume dependence

$$
\begin{aligned}
{}_{DD}{}^{i}\langle b,0,0;n,0|e^{-l(H_2+\tilde{H}_2)}|b,0,0;n,0\rangle^{i}{}_{DD} &= e^{-l\pi\frac{n^2}{2R_i^2}}4\pi^2 V_i, \\
{}_{DD}{}^{i}\langle b,0,0;n,0|e^{-l(H_2+\tilde{H}_2)}|c,0,N;n,0\rangle^{i} &= e^{-l\pi\frac{n^2}{4R_i^2}}8\pi^2 V_i, \\
{}^{i}\langle c,0,N;n,0|e^{-l(H_2+\tilde{H}_2)}|c,0,N;n,0\rangle^{i} &= e^{-l\pi\frac{n^2}{8R_i^2}}16\pi^2 V_i, \\
{}_{NN}{}^{i}\langle b,0,0;0,m|e^{-l(H_2+\tilde{H}_2)}|b,0,0;0,m\rangle^{i}{}_{NN} &= e^{-l\pi 2m^2R_i^2}\frac{4\pi^2}{V_i}, \\
{}_{NN}{}^{i}\langle b,0,0;0,m|e^{-l(H_2+\tilde{H}_2)}|c,0,0;0,m\rangle^{i} &= e^{-l\pi m^2R_i^2}\frac{8\pi^2}{V_i}, \\
{}^{i}\langle c,0,0;0,m|e^{-l(H_2+\tilde{H}_2)}|c,0,0;0,m\rangle^{i} &= e^{-l\pi\frac{m^2}{2}R_i^2}\frac{16\pi^2}{V_i},
\end{aligned}
\tag{3.3}
$$

where $V_i = (R_i)^2$. We define boundary and crosscap states $||B\rangle\rangle$ and $||C\rangle\rangle$ such that they include the volume dependence of amplitudes by zero modes:

$$
\begin{aligned}
||B\rangle\rangle_p &= \sum_{m=0}^{N-1}\Big[N^c_{b,0,m,p}|b,0,m\rangle_p(Tr\gamma_{m,p})\prod_i\frac{2\pi}{\sqrt{V_i}}\prod_j 2\pi\sqrt{V_j}\Big], \\
||C\rangle\rangle &= \sum_{m=0}^{N-1}\Big[N_{c,0,2m}|c,0,2m\rangle\prod_i\frac{4\pi}{\sqrt{V_i}}\prod_j 4\pi\sqrt{V_j}\Big], \\
||C,N\rangle\rangle &= \sum_{m=0}^{N-1}\Big[N_{c,N,2m}|c,N,2m\rangle\prod_i\frac{4\pi}{\sqrt{V_i}}\prod_j 4\pi\sqrt{V_j}\Big].
\end{aligned}
\tag{3.4}
$$

Here i and j denote compactified complex planes where the states have momentum and winding, respectively.

By using volume dependence we discuss why tadpole divergence does not cancell in Z_4, Z_8, Z_8' and Z_{12}' model. In the case of $Z_4 = \frac{1}{4}(1,1,-2)$, boundary and crosscap states with volume dependence are

$$
\begin{aligned}
||B\rangle\rangle_9 &= N_{b,0,0,9}\frac{(2\pi)^3}{\sqrt{V_1V_2V_3}}|b,0,0\rangle_9(Tr\gamma_{0,9}) + N_{b,0,1,9}|b,0,1\rangle_9(Tr\gamma_{1,9}) \\
&\quad + N_{b,0,2,9}(\frac{2\pi}{\sqrt{V_3}})|b,0,2\rangle_9(Tr\gamma_{2,9}) + N_{b,0,3,9}|b,0,3\rangle_9(Tr\gamma_{3,9}), \\
||B\rangle\rangle_5 &= N_{b,0,0,5}(\frac{2\pi}{\sqrt{V_3}})(2\pi)^2\sqrt{V_1V_2}|b,0,0\rangle_5(Tr\gamma_{0,5}) + N_{b,0,1,5}|b,0,1\rangle_5(Tr\gamma_{1,5}) \\
&\quad + N_{b,0,2,5}(\frac{2\pi}{\sqrt{V_3}})|b,0,2\rangle_5(Tr\gamma_{2,5}) + N_{b,0,3,5}|b,0,3\rangle_5(Tr\gamma_{3,5}), \\
||C\rangle\rangle &= N_{c,0,0}\frac{(4\pi)^3}{\sqrt{V_1V_2V_3}}|c,0,0\rangle + N_{c,0,2}(4\pi\sqrt{V_3})|c,0,2\rangle \\
&\quad + N_{c,0,4}(4\pi)^2\sqrt{V_1V_2}(\frac{4\pi}{\sqrt{V_3}})|c,0,4\rangle + N_{c,0,6}(4\pi\sqrt{V_3})|c,0,6\rangle, \\
||C,N\rangle\rangle &= N_{c,4,2}(4\pi\sqrt{V_3})|c,4,2\rangle + N_{c,4,6}(4\pi\sqrt{V_3})|c,4,6\rangle.
\end{aligned}
\tag{3.5}
$$

Here we omit $|c,4,0\rangle$ and $|c,4,4\rangle$ terms in $||C,N\rangle\rangle$ because these states do not contribute to the amplitude $\langle C,N|e^{-lH}|C\rangle$ by $\langle c,4,0||c,0,0\rangle = \langle c,4,4||c,0,4\rangle = 0$. $||C\rangle\rangle$ and $||C,N\rangle\rangle$ have terms with volume factor $\sqrt{V_3}$ but $||B^c\rangle\rangle_9$ and $||B^c\rangle\rangle_5$ do not. It means that tadpole from $||C\rangle\rangle$ and $||C,N\rangle\rangle$can not be cancelled by $||B^c\rangle\rangle_9$ and $||B^c\rangle\rangle_5$. So that Z_4 model is inconsistent. The same conclusion holds also for the models with orbifold group $Z_8 = \frac{1}{8}(1,3,-4), Z_8' = \frac{1}{8}(1,-3,2)$ and $Z_{12}' = \frac{1}{12}(1,5,-6)$.

In the case of Z_3 and Z_7 models, their amplitudes factorize so that tadpole cancellation condition of Z_3 and Z_7 models is reduced to massless tadpole cancellation condition in ref.[9]. We consider the case with maximal gauge symmetry in which all p-branes sit at one fixed point, for example, at the origin. From equations (2.23),(2.24) and (3.4) we determine for Z_3 model

$$\begin{aligned} ||B\rangle\rangle_9 + |C\rangle\rangle &= \frac{1}{4\sqrt{2}\pi^2}\frac{(2\pi)^3}{\sqrt{V_1V_2V_3}}\Big[|b,0,0\rangle_9(Tr\gamma_{0,9}) - 32|c,0,0\rangle\Big] \\ &+\frac{1}{4\sqrt{2}\pi^2}\Big[|b,0,1\rangle_9(Tr\gamma_{1,9}) - 4|c,0,2\rangle\Big] + \frac{1}{4\sqrt{2}\pi^2}\Big[|b,0,2\rangle_9(Tr\gamma_{2,9}) + 4|c,0,4\rangle\Big] \quad (3.6)\end{aligned}$$

Using the massless tadpole cancellation condition, we get $Tr\gamma_{0,9} = 32, Tr\gamma_{1,9} = 4$ and $Tr\gamma_{2,9} = -4$. It gives the gauge group $U(12) \times SO(8)$. In the same way we obtain for the Z_7 model, $Tr\gamma_{0,9} = 32$ and $Tr\gamma_{m,9} = 4 (m = 1 \sim 6)$ which leads to the gauge group $U(4)^3 \times SO(8)$. They are the same conclusion which is obtained by one-loop diagrams [1].

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Figures

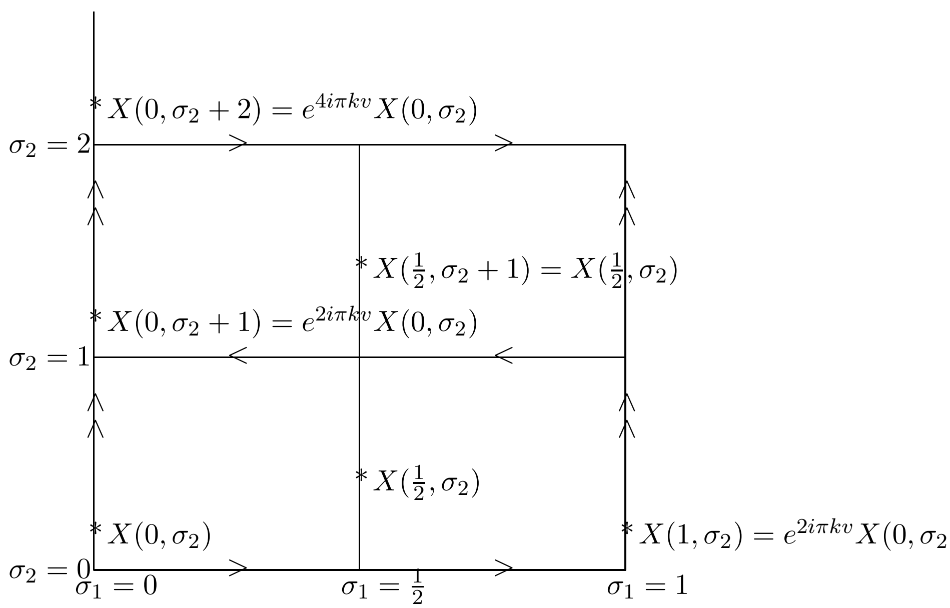

Figure 1: Klein bottle

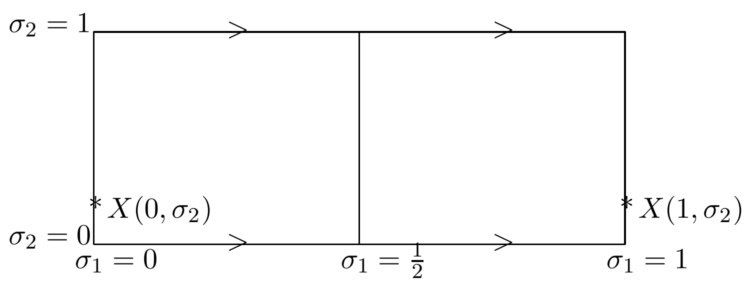

Figure 2: Cylinder

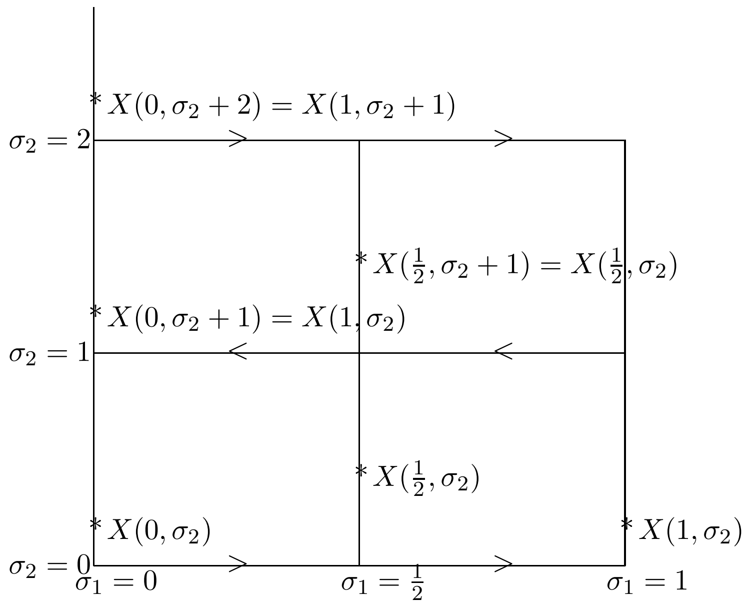

Figure 3: Möbius strip

Tables

Z_3	$\frac{1}{3}(1,1,-2)$	Z'_6	$\frac{1}{6}(1,-3,2)$	Z'_8	$\frac{1}{8}(1,-3,2)$
Z_4	$\frac{1}{4}(1,1,-2)$	Z'_7	$\frac{1}{7}(1,2,-3)$	Z'_{12}	$\frac{1}{12}(1,-5,4)$
Z_6	$\frac{1}{6}(1,1,-2)$	Z_8	$\frac{1}{8}(1,3,-4)$	Z'_{12}	$\frac{1}{12}(1,5,-6)$

Table 1: Z_N actions in $D=4$. Each vector stands for (v_1,v_2,v_3).